Additional Practice Problems for Test 2

This is a collection of problems which I prepared for tutorial. Some, but not all, were presented in tutorial. Use this to study at your own discretion, **I do not know** whether they will be similar to the difficulty of the midterm.

1. True or false?

(a) If f is differentiable then $\frac{d}{dx}\sqrt{f(x)} = f'(x)/(2\sqrt{f(x)})$ and $\frac{d}{dx}f(\sqrt{x}) = f'(x)/(2\sqrt{x})$.

(b) The function $|x|$ has a critical point at $x = 0$.

(c) The extreme values of f on $[a, b]$ are the same as the extrema of f on (a, b).

(d) If f has an absolute minimum on $[a, b]$ at c, then $f'(c) = 0$.

(e) If f is continuous and differentiable on (a, b), then f attains an absolute maximum and minimum on (a, b).

(f) If f has a local minimum at $x = c$, then $f''(c) \geq 0$.

(g) If f has a critical point at $x = c$, then c is *not* an inflection point of f.

(h) There exists a differentiable function f such that $f(1) = 0$, $f(3) = 2$, and $f'(x) > 1$ for all x. *[Hint: can you use the Mean Value Theorem?]*

(i) If f satisfies the assumptions of Rolle's Theorem, then f also satisfies the assumptions of the Mean Value Theorem.

(j) If f satisfies the assumptions of the Mean Value Theorem, then f also satisfies the assumptions of Rolle's Theorem.

2. Compute $F'(0)$ for $F(x) = \frac{x^9+x^8+4x^5-7x}{x^4-3x^2+2x+1}$. *[Hint: you can avoid computing the derivative. Write $F(x) = f(x)/g(x)$, so $F'(x) = (f'g - fg')/g^2$, and plug in x right away.]*

3. What is the 30$^{\text{th}}$ derivative of $(2x^6 + x^4)^5$?

4. What is the 50$^{\text{th}}$ derivative of $\cos(2x)$?

5. What is the derivative of $|x|$? *[Hint: write $|x| = \sqrt{x^2}$ and use the chain rule. Why can we write $|x| = \sqrt{x^2}$?]*

6. Suppose that f, g, and h are differentiable functions. Compute the derivative of

$$\left(f(x^2)\right)^{10} \frac{e^{g(|\sin x|)}}{h(\ln x)}.$$

7. At what point(s) on the curve $y = \sqrt{1 + 2x}$ is the tangent line to y perpendicular to the line $6x + 2y = 1$? *[Recall that lines are perpendicular if their slopes are negative reciprocals at the point of intersection.]*

8. Suppose that f and g are differentiable functions such that $f(g(x)) = x$ and $f'(x) = 1+(f(x))^2$. What is $g'(x)$?

9. Compute dx/dy, the derivative of x with respect to y, for the curve $y \sec x = x \tan y$.

10. Let P be a point on the circle $x^2 + y^2 = r^2$. Show that the tangent line to the circle at P is perpendicular to the radius from the origin to P. *[Recall that lines are perpendicular if their slopes are negative reciprocals at the point of intersection.]*

11. Use the definition of the derivative to compute $f'(0)$ for the function

$$f(x) = \begin{cases} x^2 \arctan(1/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

12. Find the critical points of $f(x) = |x^3 + x^2 - 16x - 16|$.

13. Find the domain and the derivative of $f(x) = \ln(\ln(\ln(g(x))))$.

14. Differentiate $y = \dfrac{x^{3/4}\sqrt{x^2+1}}{(3x+4)^5}$. *[Hint: can you avoid the quotient rule?]*

15. For which of the following functions does Rolle's Theorem guarantee the existence of a point with slope 0 on the given interval? If not, what about a smaller interval?

(a) $x^{1002} - x^{1000} + x^2 - 1$ on $[-1, 1]$

(b) $|x|$ on $[-1, 1]$

(c) $\dfrac{x(x-1)(x-2)}{x-1}$ on $[0, 2]$

(d) $x(x-8)$ on $[0, 10]$

16. Sketch a continuous graph with roots at -2 and 1 and critical points at -1 and 3. Is it unique?

17. Find all local maxima and minimum of $f(x) = x - 2\sin x$ on the interval $[0, 2\pi]$.

18. Show that $f(x) = x^3 - 2x^2 + 2x$ is an increasing function.

19. Sketch the curve $f(X) = 5x^3 - 3x^5$. This includes:

- find the intervals of increase and decrease,
- find the local and global extrema
- find the intervals of concavity.

20. Sketch the graph of a function defined on $[0, 8]$ with $f(0) = f(8) = 0$ that does not satisfy the conclusion of Rolle's Theorem on $[0, 8]$.

21. Let $f(x) = 2 - |2x - 1|$. Show that there is no value of $c \in (0, 3)$ such that $f(3) - f(0) = f'(c)(3 - 0)$. Why does this not contradict the Mean Value Theorem?

22. Assume that f is twice differentiable, positive, and concave up. Show that $g(x) = (f(x))^2$ is also positive and concave up. What's an example of such a function f?

Challenge questions

C1. (a) Show that the derivative of an even function is odd.

(b) Show that the derivative of an odd function is even.

C2. Show that $\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'(f^{-1}(x))}$.

C3. Suppose that f is continuous on $[a, b]$, differentiable on (a, b), and has a local minimum at $c \in (a, b)$. Show that $f'(c) = 0$.