Math 1XX3 Tutorial Problems

for T04, T07 with Mike

Tutorial 11/Week 12

Topics: Gradient. Directional derivatives.

Note: Solutions to these problems will be posted at the end of the week.

1. True or false? Justify your answer. If $f(x, y) = \sin x + \sin y$, then $-\sqrt{2} \leq D_{\mathbf{u}} f(x, y) \leq \sqrt{2}$ for any unit vector $\mathbf{u}$.

2. You are standing at the point $(x, y, z) = (0, 3, 9)$ on a mountain whose elevation at (x, y) is given by
$$h(x, y) = \frac{y^2}{x^2 + 1},$$
where the positive x-axis points east, and the positive y-axis points north. In which direction (e.g., west, northeast, etc.) could you walk in order to initially maintain your current elevation?

3. Find all points at which the direction of fastest change of the function $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\hat{\imath} + \hat{\jmath}$.

4. Find the directions in which the directional derivative of $f(x, y) = x^2 + xy^3$ at the point $(2, 1)$ has the value 2.

5. The **second directional derivative** of $f(x, y)$ is
$$D_{\mathbf{u}}^2 f(x, y) = D_{\mathbf{u}} \left[D_{\mathbf{u}} f(x, y) \right].$$
If $\mathbf{u} = \langle a, b \rangle$ is a unit vector and f has continuous second partial derivatives, show that
$$D_{\mathbf{u}}^2 f = a^2 f_{xx} + 2ab f_{xy} + b^2 f_{yy}.$$