Math 1XX3 Tutorial Problems

for T04, T07 with Mike

Tutorial 1/Week 2

Topics: Properties and solutions of differential equations. Separable equations. Slope fields.
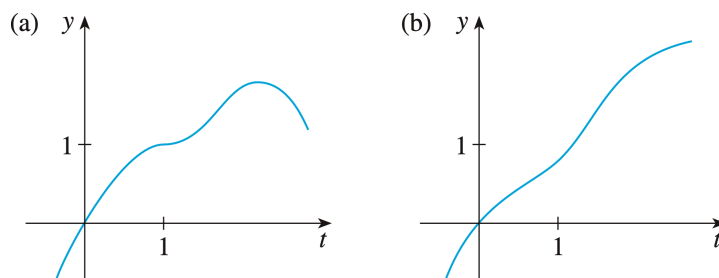
1. True or false?

   (a) All solutions of the differential equation $y' = -1 - y^4$ are decreasing functions.

   (b) The equation $y' = 3y - 2x + 6xy - 1$ is separable.

2. Explain why the following graphs *cannot* be solutions of the differential equation

$$\frac{dy}{dt} = e^t(y-1)^2.$$

3. (a) What can you say about the graph of a solution of the equation $y' = xy^3$ when x is close to 0? What if x is large?

   (b) Verify that all members of the family $y = (c - x^2)^{-1/2}$ are solutions of the differential equation $y' = xy^3$.

   (c) Solve the initial value problem $y' = xy^3$ when $y(0) = 2$.

4. (a) Match the differential equations to their direction field below.

Differential Equation	Direction Field
$y' = 2\sin x$	
$y' = x - y$	
$y' = y^2 - x^2$	
$y' = 1 - x + y - xy$	

Direction field (i)

Direction field (ii)

Direction field (iii)
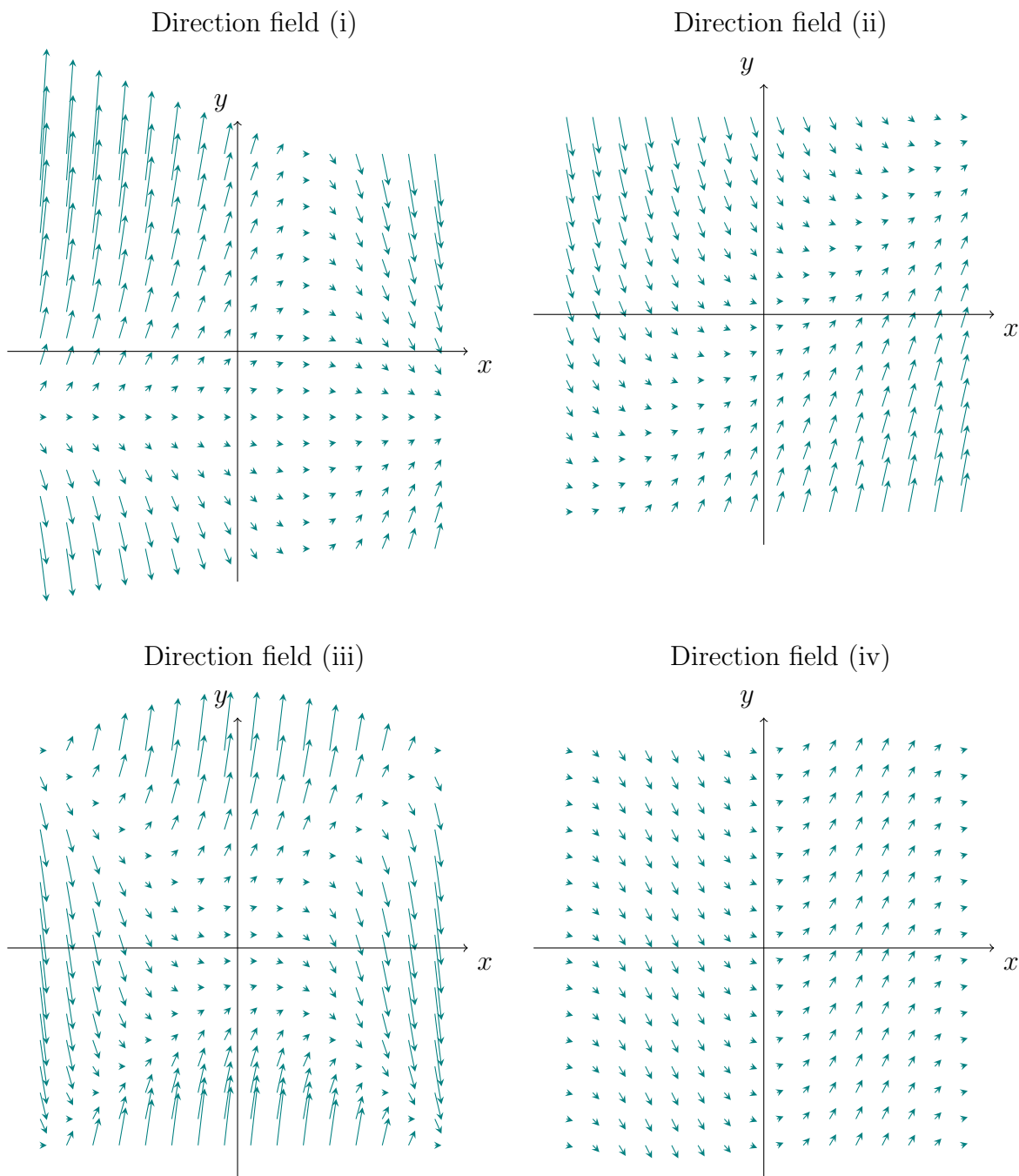
Direction field (iv)

(b) Find a solution for the differential equation corresponding to the direction field (i) with initial value $y(0) = 0$.

5. If y is the solution to the initial value problem

$$\begin{cases} \frac{dy}{dt} = 2y\left(1 - \frac{y}{5}\right), \\ y(0) = 1, \end{cases}$$

find $\lim_{t \to \infty} y$. What happens to the limit as we vary the initial value?