Math 1XX3 Tutorial Problems

for T04, T07 with Mike

Tutorial 3/Week 4

Topics: Sequences, series, and convergence. Monotone Sequence Theorem. Alternating Series Test. Theorem 1 in Section 10.4

1. True or false?

   (a) If the sequences $\{a_n\}$ and $\{b_n\}$ are divergent, then the sequence $\{a_n + b_n\}$ is divergent.

   (b) If $\sum c_n 6^n$ is convergent, then $\sum c_n(-6)^n$ is convergent.

   (c) If $\lim_{n\to\infty} a_n = 2$, then $\lim_{n\to\infty}(a_{n+3} - a_n) = 0$.

   (d) If $\sum a_n$ is divergent, then $\sum |a_n|$ is divergent.

2. Which of the following sequences converge? Explain. For those that do, find the limit.

   (a) $a_n = \arctan(\sqrt{n^5 + n^4 + n^3 + n^2 + n})$ (b) $b_n = \ln\left(1 + \frac{1}{n}\right)$

3. Which of the following series converge? Explain.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{6 \cdot 2^{2n-1}}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right)$

   [Hint for (b). Is the series telescoping?]

4. Consider the series $\sum(a_n + b_n)$. Do there exist a_n and b_n such that

   $$\sum_{n=1}^{\infty}(a_n + b_n) \neq \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n,$$

   i.e., we cannot split over the sum? If so, give an example of a_n and b_n and explain why this doesn't contradict Theorem 1 in section 10.2. If no such a_n and b_n exist, why not?

5. Find the values of x for which the series converges. For these values of x, find the sum.

   $$\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$$