Math 1XX3 Tutorial Problems

for T04, T07 with Mike

Tutorial 4/Week 5

Topics: Q & A and review for Test 1. Convergence of series. Power series.

Note: In tutorial, before we go through the following exercises, we will have a question-and-answer session. You can submit questions ahead of time on this Google Form. Your questions can be about the practice test or any other course content/theorem/technique/etc.

1. Let's make "toolboxes", a list of methods that we commonly use to solve questions. List the techniques that you could use to...

   (a) Solve a differential equation.

   (b) Determine the convergence/divergence of a series.

2. What approach would you use to determine whether the following series converge (absolutely or conditionally) or diverge?

   (a) $\displaystyle\sum_{n=1}^{\infty} \cos(1/n)$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)3^n}{2^{2n-1}}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^4 - 3n^2 + 17}{n^5 - 3n^3 + 19}\right)^n$

3. Use series to evaluate the following limit.

$$\lim_{x\to 0} \frac{\sin x - x}{x^3}$$

4. True or false? If true, provide a brief justification. If false, provide a counterexample.

   (a) If $\sum a_n$ and $\sum b_n$ are divergent, then $\sum a_n b_n$ is divergent.

   (b) The series $\sum n^{-\sin 1}$ is divergent.

   (c) The ratio test can be used to determine whether the series $\sum 1/n!$ and $\sum 1/n^3$ are convergent.