Math 1XX3 Tutorial Problems

for T04, T07 with Mike

Tutorial 6/Week 7

Topics: Parametric equations, arc length, speed, polar coordinates.

1. True or false? Briefly justify your answer.
   (a) If the parametric equation $c(t) = (f(t), g(t))$ satisfies $g'(1) = 0$, then it has a horizontal tangent when $t = 1$.
   (b) The parametric curves $c_1(t) = (t^2, t^4)$ and $c_2(t) = (t^3, t^6)$ have the same graph.

2. Find the cartesian equation of the parametrized curve $c(t) = (e^t, t^2)$.

3. Denote by $\mathcal{C}$ the curve $x^2 + y^2 = 1$ where we restrict to $x \geq 0$. That is, $\mathcal{C}$ is the right-half of the unit circle.
   (a) Find a parametrization $c(t)$ of $\mathcal{C}$ where $t \in [-1, 1]$ and travels counterclockwise. That is, find a parametrization $c(t)$ with endpoints $c(-1) = (0, -1)$ and $c(1) = (0, 1)$.
   (b) Calculate the speed function of $c(t)$.
   (c) Compute the arclength function for $c(t)$.

   [Hints: For the first part, don't use polar coordinates. For the arclength, remember that $\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$*.]*

4. Match each polar coordinates equation with its corresponding cartesian representation.

   (a) $r = 2$, (i) $x + y = 4$,
   (b) $r = 2\sin\theta$, (ii) $x^2 + y^2 = 4$,
   (c) $r^2(1 - 2\sin^2\theta) = 4$, (iii) $x^2 - y^2 = 4$,
   (d) $r(\cos\theta + \sin\theta) = 4$, (iv) $x^2 + (y-1)^2 = 1$.

5. **Bonus.** *Viviani's Curve* is a curve in $\mathbb{R}^3$ which is given by the intersection of the unit sphere centered at $\left(-\frac{1}{2}, 0, 0\right)$ and the circular cylinder of radius $\frac{1}{2}$ centered about the z-axis. Show that Viviani's Curve is parametrized by $c(t) = \left(\cos^2 t - \frac{1}{2}, \sin t\cos t, \sin t\right)$.

   [Hints: the equation of this cylinder is $x^2 + y^2 = 1/4$ and the equation of this sphere is $(x + 1/2)^2 + y^2 + z^2 = 1$. A point is on the curve if it satisfies both of these equations. Start with the z coordinate: notice that $-1 \leq z \leq 1$ if it is on the curve so choose $z = \sin t$. Then derive the x coordinate, then the y coordinate.]