Math 1XX3 Tutorial Problems

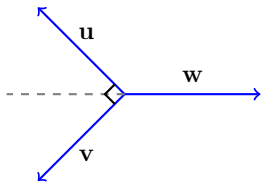
for T04, T07 with Mike

Tutorial 7/Week 8

Topics: Vectors in the plane. 3D surfaces, vectors, curves. Dot product and angle.

1. True or false? Briefly justify your answers. Let $\mathbf{u}$ and $\mathbf{v}$ be any two vectors in $\mathbb{R}^2$.
   (a) If $\mathbf{u} \cdot \mathbf{v} = 0$, then $\mathbf{u} = 0$ or $\mathbf{v} = 0$.
   (b) $|\mathbf{u} \cdot \mathbf{v}| = |\mathbf{u}||\mathbf{v}|$.
   (c) If k is a scalar (any real number), then $k(\mathbf{u} \cdot \mathbf{v}) = (k\mathbf{u}) \cdot \mathbf{v}$.

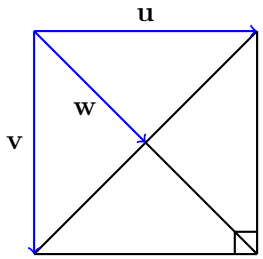
2. If the vectors in the figure satisfy $|\mathbf{u}| + |\mathbf{v}| = 1$ and $\mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0}$, what is $|\mathbf{w}|$?



3. (a) Find an equation of the sphere that passes through the point $(6, -2, 3)$ and has center $(-1, 2, 4)$.
   (b) Find an equation of the cylinder which passes through the point $(6, -2, 3)$ and has center $(-1, 2, 4)$ and whose central axis is the vertical axis.
   (c) Find the center and radius of the following sphere.

$$x^2 + y^2 + z^2 - 8x + 2y + 6z + 1 = 0$$

4. Suppose $\mathbf{u}$ is a unit vector as pictured to the right.
   (a) Find $\mathbf{u} \cdot \mathbf{v}$ and $\mathbf{u} \cdot \mathbf{w}$.
   (b) Calculate $\mathbf{u} + \mathbf{v} + \mathbf{w}$. Argue geometrically about the value of $\mathbf{u} \cdot (\mathbf{u} + \mathbf{v} + \mathbf{w})$.
   (c) Now compute $\mathbf{u} \cdot (\mathbf{u} + \mathbf{v} + \mathbf{w})$. Does your geometric argument line up with your computation?



5. Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$ be nonzero vectors. Show that $\mathbf{v} - \mathbf{u}_{||\mathbf{v}}$ is orthogonal to $\mathbf{u}$. *[Hint: dot product.]*