Math 3GR3, Tutorial 2

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Tutorial problems

Topics: SageMath. Groups, Cayley tables, commutativity.

Example 1. Course webpage: https://math.mcmaster.ca/~matt/3gr3/index.html.

- Use the Sage cell on the course webpage
- Open online version of the course textbook
- Enter the following commands:

```
a = 11
b = 77115025
gcd(a, b)
>> run cell

# Q: what does the following output give us?
xgcd(a, b)
```

For fun:

```
for g in graphs(4):

    if not g.is_connected():
        continue

    g.show()
    print('\n')
```

Question 2. Which of the following Cayley tables form a group?

(a) [Judson Exercise 3.5.2(a)]

$\circ$	a	b	c	d
a	a	c	d	a
b	b	b	c	d
c	c	d	a	b
d	d	a	b	c

(b)

	e	w	x	y	z
e	e	w	x	y	z
w	w	e	y	z	x
x	x	z	e	w	y
y	y	x	z	e	w
z	z	y	w	x	e

Question 3. Compute the Cayley tables of the following additive groups:

(a) $\mathbb{Z}_4$, (b) $\mathbb{Z}_2 \times \mathbb{Z}_2$.

Question 4 (Judson Exercise 3.5.7)**.** Let $S = \mathbb{R} \setminus \{-1\}$ and define a binary operation on S by $a * b = a + b + ab$. Prove that $(S, *)$ is an abelian group.

Question 5 (Judson Exercise 3.5.32)**.** Let G be a group with a finite and even number of elements. Show that there exists some *nonidentity* $a \in G$ such that $a^2 = e$.