Math 3GR3, Tutorial 4

Mike Cummings

October 3, 2023

Topics: Cyclic groups. Permutation groups. Cycles.

Question 1. Recall that subgroups of a cyclic group are cyclic.
True or false? Fix an integer $n > 1$. Since $\mathbb{Z}$ is cyclic, so is $U(n)$.

Question 2. Let p be prime and r a positive integer. What are the generators of $\mathbb{Z}_{p^r}$? How many are there?

Question 3. Compute A^{1223} for the permutation $A \in S_9$ given by

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 1 & 4 & 5 & 4 & 7 & 8 & 9 & 6 \end{pmatrix}.$$

Question 4 (Judson 3.5.35)**.** Find all the subgroups of the symmetry group of an equilateral triangle.

Question 5. Let H be a subgroup of a group G and fix some $g \in G$. Show that gHg^{-1} is also a subgroup of G.

$$gHg^{-1} = \{ghg^{-1} \mid h \in H\}.$$

Question 6. Fix a subgroup $H = \{id, \rho_1, \rho_2\}$ of the group of symmetries of the equilateral triangle. Compute $\mu_1 H \mu_1^{-1} = \mu_1 H \mu_1$.

Question 7. Let G be an abelian group of order pq with elements a and b of orders p and q, respectively. If $\gcd(p, q) = 1$, then show that G is cyclic.