

# Math 3GR3, Tutorial 5

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October 17, 2023

**Topics:** Alternating groups, dihedral groups.

**Question 1** (Judson 5.4.9). Does  $A_8$  contain an element of order 26?

**Question 2** (Judson 5.4.33). Suppose a permutation  $\alpha$  satisfies  $\alpha\beta = \beta\alpha$  for all  $\beta \in S_n$ . Show that  $\alpha$  must be the identity.

**Question 3** (Judson 5.4.34). If  $\alpha$  is even, show that  $\alpha^{-1}$  is too. Does the corresponding result hold if  $\alpha$  is odd?

**Question 4** (Judson 5.4.37). Let  $r$  and  $s$  be a rotation and reflection in  $D_n$ . Show that  $srs = r^{-1}$  and that  $r^k s = sr^{-k}$ .

**Question 5** (Judson 5.4.5). Find each of the following sets. Are any of these sets subgroups of  $S_4$ ?

(a)  $A = \{\sigma \in S_4 \mid \sigma(1) = 3\}$

(b)  $B = \{\sigma \in S_4 \mid \sigma(2) = 2\}$

(c)  $C = \{\sigma \in S_4 \mid \sigma(1) = 3 \text{ and } \sigma(2) = 2\}$