Math 3GR3, Tutorial 6

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October 24, 2023

Topics: Subgroups of the symmetric group. Cosets. Lagrange's Theorem.

Question 1 (Judson 6.5.5)**.** Describe the left and right cosets of

(a) $\langle 3 \rangle$ in $U(8)$,

(b) D_4 in S_4,

(c) A_n in S_n for all n.

Question 2 (Judson 6.5.17)**.** Suppose that $[G : H] = 2$. If a and b are not in H, show that $ab \in H$.

Question 3 (Judson 6.5.16)**.** If $|G| = 2n$, prove that the number of elements of order 2 is odd. Use this result to show that G must contain a subgroup of order 2.

Question 4 (Judson 5.4.5)**.** Write out the elements of the following subset of S_4 (e.g., in permutation notation). Is it a subgroup of S_4?

$$S = \{\sigma \in S_4 \mid \sigma(1) = (3)\}.$$