Math 3GR3, Tutorial 7

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Topics: Cosets partition a group. Isomorphisms.

Question 1. Partition the group G of symmetries of a triangle by left cosets of $H = \{e, \mu_1\}$. Recall that the Cayley table for G is as follows.

$\circ$	e	ρ_1	ρ_2	μ_1	μ_2	μ_3
e	e	ρ_1	ρ_2	μ_1	μ_2	μ_3
ρ_1	ρ_1	ρ_2	e	μ_3	μ_1	μ_2
ρ_2	ρ_2	e	ρ_1	μ_2	μ_3	μ_1
μ_1	μ_1	μ_2	μ_3	e	ρ_1	ρ_2
μ_2	μ_2	μ_3	μ_1	ρ_2	e	ρ_1
μ_3	μ_3	μ_1	μ_2	ρ_1	ρ_2	e

With this example as motivation, let us review Lemma 6.3.

Lemma 6.3. Let H be a subgroup of G and pick $g_1, g_2 \in G$. The following are equivalent.

(i) $g_1 H = g_2 H$

(ii) $H g_1^{-1} = H g_2^{-1}$

(iii) $g_1 H \subset g_2 H$

(iv) $g_2 \in g_1 H$

(v) $g_1^{-1} g_2 \in H$

For instance, in the above example, $\mu_3 H = \rho_1 H$ since $\mu_3 \in \rho_1 H$.

Question 2 (Judson 6.5.8)**.** Prove that $\mathbb{Q}$ is not isomorphic to $\mathbb{Z}$.

Question 3 (Judson 9.4.7)**.** Show any cyclic group G of order n is isomorphic to $\mathbb{Z}_n$.

Question 4 (Judson 9.4.2)**.** Let G be the subgroup of $\mathbf{GL}_2(\mathbb{R})$ consisting of matrices of the following form.

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

Show that $G \cong \mathbb{C}^*$.