Math 2LA3 Assignment 1

1. Consider the following linear programming problem.
   (a) Sketch the feasible region.
   (b) List the vertices of the feasible region.
   (c) What is the maximal value of the objective function? At what point does it occur?

2. Consider the feasible region in $\mathbb{R}^2$ associated to the following constraints:

$$\begin{cases} \alpha x + \beta y \leq b_1 \\ \gamma x + \delta y \leq b_2 \\ x, y \geq 0, \end{cases}$$

   where $\alpha, \beta, \gamma, \delta$, and b_1, b_2 are any integers.
   (a) What is the minimum number of possible vertices for the feasible region?
   (b) Give an example of a set of constraints of the above form that attains this minimum and justify your answer.

3. Use a 3D graphing calculator to find the vertices of the feasible region corresponding to the following set of constraints. Submit a screenshot of your feasible region where you have clearly labelled the vertices.

   For the sake of clarity, it may be clearer to replace the inequalities with equalities in the graphing calculator.

4. Give an example of an optimization problem that is not a linear programming problem. Briefly justify your answer.

5. For the following linear programming problem, set up the corresponding initial simplex tableau. You do not need to solve the linear programming problem.

6. Given the following simplex tableau, which entry should be the next pivot? (No justification required.)

7. Use the simplex method to solve the following linear programming problem.

   A furniture company manufactures chairs, beds, and tables with the aid of workers A, B, and C. The time (in hours) that each worker spends on each of these items and the corresponding profits are shown in the following table.

	A	**B**	**C**	**Profit ($)**
Chair	5	0	3	15
Bed	3	6	0	20
Table	0	3	4	30

   Employees A and B can each work at most 40 hours per week, while employee C can work at most 30 hours per week. How many chairs, beds, and tables should be built in order to maximize profit?

   For this question, we are allowed to build a fraction of a piece of furniture. So, for example, if the profit is maximized when you build 2.1 chairs, you need not round to 2 chairs.