Math 2LA3 Assignment 2

1. Use the simplex method to solve the following **non-canonical** linear programming problem.

$$\begin{aligned} &\texttt{minimize} && 3x + y + 2z \\ &\texttt{subject to} && \begin{cases} x + 2y + 3z \geq 24 \\ 2x + 4y + 3z = 36 \\ x, y, z \geq 0 \end{cases} \end{aligned}$$

2. Let A be an $n \times n$ matrix and consider the following statements.

   (i) If A has n distinct eigenvalues, then A is diagonalizable.

   (ii) If A is diagonalizable, then A has n distinct eigenvalues.

   Determine whether each of the above statements are true or false.

   (a) both (i) and (ii) are true

   (b) (i) is true and (ii) is false

   (c) (i) is false and (ii) is true

   (d) both (i) and (ii) are false

3. Let N be the last digit of your student number. Find all real numbers a and b such that the following matrix is **not** diagonalizable. Justify your answer.

$$X = \begin{bmatrix} N & a \\ 0 & b \end{bmatrix}$$

4. Give an example of a nondiagonal 2×2 matrix that is diagonalizable but not invertible. Justify your answer.

5. Diagonalize the following matrix. Show all your work.

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

6. Let ℓ and m be real numbers. Solve the following differential equation.

$$\begin{cases} y'(t) = \ell y(t), \\ y(0) = m. \end{cases}$$

7. Suppose that populations of alderflies, bears, and carp are modelled over time by functions $a(t)$, $b(t)$, and $c(t)$, respectively, that satisfy the differential equations

$$\begin{cases} a'(t) = 10a(t) + 6b(t) + c(t) \\ b'(t) = -14a(t) - 9b(t) - 2c(t) \\ c'(t) = -8a(t) - 4b(t) - c(t). \end{cases}$$

   Find $a(t)$, $b(t)$, and $c(t)$ if we have initial populations of $a(0) = 1000$, $b(0) = 12$, and $c(0) = 128$.